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Finding the optimal opening time of harvesting farmed fishery resources

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Abstract

As an application of mathematics to engineering problems, this paper formulates a simple optimal stopping problem to decide the opening time of harvesting farmed fishery resources that maximizes an economic objective function. A sufficient condition for unique existence of the internal optimal opening time is provided and its concrete mathematical analysis is carried out. Comparative statics of the optimal opening time clearly reveals its dependence on the parameters of the farming environment. The problem is finally applied to analyzing management of a commercially important fishery resource in Japan.

Keywords: Aquaculture, Optimal stopping problem, Population dynamics, Harvesting, Optimal opening time

1 Introduction

Aquacultures farm aquatic organisms for commercial purposes and their production value has been rapidly growing over the world [5]. Establishment of appropriate management policies of farmed aquatic organisms from economic viewpoints has been an urgent issue for current fishery sectors, which would depend on type and purpose of each aquaculture [7, 12, 13, 17, 19]. One of the most crucial issues in operating aquacultures is to decide the optimal opening time of harvesting the farmed fishery resources after which they are harvested and sold; however, the opening time has empirically been determined in the conventional aquacultures.

In order to approach this issue from a theoretical side, this paper considers a new simple and deterministic, but nontrivial optimal stopping (starting) problem [2, 11] on management of farmed aquatic organisms in an aquaculture system. The goal of the problem is finding an economically optimal opening time of harvesting farmed fishery resources, after which they are harvested with a known intensity and sold. The problem reduces to finding a solution (the optimal opening time) to differential equations whose behavior can be analytically resolved. This paper presents a sufficient condition for unique existence of an internal solution, its comparative statics, and a real application focusing on a current Japanese aquaculture system.

The rest of this paper is organized as follows. Section 2 presents the governing equation of the population dynamics of farmed aquatic organisms as the fishery resources. The optimal stopping problem is formulated in the same section. Section 3 performs mathematical analysis of the problem with the particular emphasis on the unique existence of the optimal opening time and its comparative statics. An application of the problem to the commercially important fish *Plecoglossus altivelis* (*P. altivelis*) in Japan [8, 15, 18] is also performed in this section. Section 4 concludes this paper.

2 Mathematical formulation

2.1 Population dynamics

The period of farming homogenous aquatic organisms in an aquaculture system (a pool) is the interval $[0,T]$ with a fixed terminal time T. The time is denoted by t. The farming starts and ends at $t = 0$ and $t = T$, respectively. The total number of individuals and the individual body weight in the system are denoted as $N_t : [0,T] \to [0,+\infty)$ and $W_t : [0,T] \to [0,+\infty)$, respectively. Given initial conditions $N_0 > 0$ and $W_0 > 0$, the governing ordinary differential equations (ODEs) of N_t and W_t are specified as

$$\frac{\mathrm{d}N_t}{\mathrm{d}t} = -\left(R + c\chi_{\{t>\tau\}}\right)N_t \quad \text{and} \quad \frac{\mathrm{d}W_t}{\mathrm{d}t} = f(W_t) = W_t g(W_t), \tag{1}$$

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respectively where $R > 0$ is the natural mortality, $c > 0$ is the harvesting rate, and $f, g \in C^1(\mathbb{R})$ are chosen so that W_t with a sufficiently small $W_0 > 0$ is increasing and bounded for $t \in [0, T]$ (Proposition B.7 of Smith and Waltman [14]). τ represents the opening time of harvesting the aquatic organisms to be optimized and χ_S is the usual indicator function for the set S. Note that the governing ODE of W_t, which is a source of nonlinearity of the optimal stopping problem to be presented in this paper, is independent of τ and is decoupled with that of N_t.

2.2 Objective function

The admissible range of τ is $[0, T]$. The objective function J_τ to be maximized with respect to τ is set as

$$J_\tau = \alpha \int_\tau^T cN_s W_s \mathrm{d}s - \beta \int_0^T pN_s W_s \mathrm{d}s \tag{2}$$

where $\alpha > 0$ and $\beta > 0$ are weight parameters and $p > 0$ is the unit-time cost of farming, such as for feeding the organisms and for cleaning up their excrements in the pool. The right side of (2) is a sum of the profit by the harvesting during (τ, T) (first term) and the cost of the farming during the whole period $(0, T)$ (second term). The objective function (2) is qualitatively different from the conventional ones with singular control variables having deltaic harvesting strategies that harvest the whole population at once [9, 10, 12]. The present objective function rather considers a situation where the population is continuously harvested in time with the known or predicted harvesting rate c and the cost of harvesting is small and negligible. Actually, such a situation is common in actual management of farmed *P. altivelis* with small-scale aquaculture systems in Japan as focused on later.

The objective function J_τ can be rewritten as

$$I_\tau \equiv \frac{1}{N_0 \beta p} J_\tau = -\int_0^\tau e^{-Rs} W_s \mathrm{d}s + (\gamma - 1) e^{-R\tau} \int_\tau^T e^{-(R+c)(s-\tau)} W_s \mathrm{d}s \tag{3}$$

with the derivatives

$$\frac{\mathrm{d}I_\tau}{\mathrm{d}\tau} = \gamma e^{-R\tau} (-W_\tau + K_\tau) \tag{4}$$

and

$$\frac{\mathrm{d}^2 I_\tau}{\mathrm{d}\tau^2} = \gamma e^{-R\tau} \left(c(K_\tau - W_\tau) + \left(R + \frac{c}{\gamma} - g(W_\tau) \right) W_\tau \right) \tag{5}$$

where K_τ is given by

$$K_\tau = c \left(1 - \frac{1}{\gamma} \right) \int_\tau^T e^{-(R+c)(s-\tau)} W_s \mathrm{d}s \in C([0,T]) \cap C^1(0,T) \tag{6}$$

and $\gamma = \frac{\alpha c}{\beta p}$. The condition $\gamma > 1$ for the situation where the profit of the harvesting exceeds the cost of farming per unit time is assumed in this paper. Maximizing J_τ with respect to τ is equivalent to doing so for I_τ. It is straightforward to show that K_τ in (6) solves the terminal value problem of the ODE

$$\frac{\mathrm{d}K_\tau}{\mathrm{d}\tau} = (R + c) K_\tau - c \left(1 - \frac{1}{\gamma} \right) W_\tau \text{ for } 0 \le \tau < T \text{ with } K_T = 0. \tag{7}$$

Derivation procedures of (4), (5), (6), and (7) are presented in Appendix. According to (4) and the classical theory of statistic optimization problems (Chapter 2.2 of Bonnans et al. [1]), the optimal opening time $\tau = \tau^*$ assuming it is an internal solution $\tau^* \in (0, T)$ has to satisfy

$$W_{\tau^*} = K_{\tau^*} \text{ and } \frac{\mathrm{d}^2 I_{\tau^*}}{\mathrm{d}\tau^2} = \gamma e^{-R\tau^*} W_{\tau^*} \left(R + \frac{c}{\gamma} - g(W_{\tau^*}) \right) < 0 \tag{8}$$

where the first equation of (8) is the necessary condition for an extreme value and the second one is that to be a local maximum. Notice that $I \in C([0, T]) \cap C^2(0, T)$.

Remark 2.1 Mathematically, β and p can be combined into single parameter; however, each variable has different industrial meaning: the weight and the cost. They are separately described in this paper due to the abovementioned reason.

Remark 2.2 The present formulation and the mathematical analysis below are still valid for a stochastic counterpart where W_t follows a diffusion process and its mean $\mathrm{E}[W_s]$ is known explicitly like the stochastic Gompertz model [4, 21]. In this case, I_τ is replaced by

$$\mathrm{E}[I_\tau] = -\int_0^\tau e^{-Rs} \mathrm{E}[W_s] \mathrm{d}s + (\gamma - 1) e^{-R\tau} \int_\tau^T e^{-(R+c)(s-\tau)} \mathrm{E}[W_s] \mathrm{d}s. \tag{9}$$

Considering the effects of uncertain model parameters in the governing ODE of W_t as in Dorini et al. [3] can also be possible based on (9), which will be addressed in forthcoming papers.

3. Mathematical analysis

This section presents concrete mathematical analysis results on unique existence and comparative statics on τ^* with concise proofs. In what follows, ε represents a sufficiently small positive constant whose value depends on the context.

Firstly, assuming the simplest case $g(W_t) = r = const > 0$, the following proposition holds.

Proposition 3.1: *For* $g(W_t) = r = const > 0$ *with* $r > R + c$, τ^* *is analytically expressed as*

$$\tau^* = \max\left\{0, T - \frac{1}{r-(R+c)} \ln\left(1 + \frac{\gamma}{\gamma-1}\frac{r-(R+c)}{c}\right)\right\}. \tag{10}$$

Based on Theorem 6.4 of Thieme [16], additional assumptions for more realistic, sigmoid-like growth of individuals are specified.

Assumption 3.2 *f is sufficiently regular so that* $f \in C^2(\mathbb{R})$. *There exists one* $a > 0$ *such that* $f(0) = f(a) = 0$, $f(w) > 0$ *for* $0 < w < a$, *and* $f''(w) < 0$ *for* $0 < w < a$. *In addition, there exists L such that* $0 < L < a$, $f'(w) > 0$ *for* $0 < w < L$, $f'(w) < 0$ *for* $L < w < a$, *and* $f'(L) = 0$. *Furthermore,* $0 < W_0 < a$.

Assumption 3.2 leads to $W \in C([0,T]) \cap C^2(0,T)$, $0 < W_t < a$ with $\frac{dW_t}{dt} > 0$ for $0 < t < T$ and W_t has at most one inflection point for $0 < t < T$ where $\frac{d^2 W_t}{dt^2}$ changes the sign from positive to negative. It also leads to $g(w) > 0$ and $g'(w) < 0$ for $0 < w < a$.

Notice that $W_T > K_T = 0$ and $W, K \in C([0,T]) \cap C^1(0,T)$. Therefore, at least one $\tau^* \in (0,T)$ exists if

$$W_0 < K_0 = c\left(1 - \frac{1}{\gamma}\right)\int_0^T e^{-(R+c)s} W_s \mathrm{d}s \tag{11}$$

by the classical mean value theorem, which is valid at least for sufficiently large *g*: namely, for fishery resources that grow well. The next lemma on the profile of K_τ is used for proving unique existence of the internal τ^*.

Lemma 3.3: K_τ *has no local minimum and has at most one local maximum for* $0 < \tau < T$ *at some* $\tau = \tau_0$. In addition, $K_\tau < W_\tau$ *for* $\tau_0 \le \tau \le T$.

(Proof of Lemma 3.3) Since $K \in C([0,T]) \cap C^2(0,T)$. If K_τ has a local minimum at a $\tau_0 \in (0,T)$, then $\frac{dK_{\tau_0}}{d\tau} = 0$. It is straightforward to show

$$\frac{\mathrm{d}^2 K_{\tau_0}}{\mathrm{d}\tau^2} = -c\left(1 - \frac{1}{\gamma}\right) g(W_{\tau_0}) W_{\tau_0} < 0. \tag{12}$$

Hence the local extreme cannot be a local minimum, showing that K_τ has no local minimum and therefore has at most one local maximum for $0 < \tau < T$ because of its smoothness. By (7), assuming K_{τ_0} is a local maximum yields

$$K_{\tau_0} = \frac{c}{R+c}\left(1 - \frac{1}{\gamma}\right) W_{\tau_0} < W_{\tau_0}, \tag{13}$$

which with $\frac{dK_\tau}{d\tau} < 0$ and $\frac{dW_\tau}{d\tau} > 0$ for $\tau_0 < t \le T$ completes the proof.

Lemma 3.3 then leads to the following theorem, which is the main result of this paper.

Theorem 3.4: $\tau^* \in (0,T)$ *uniquely exists under* ***Assumption 3.2*** *and* (11).

(Proof of Theorem 3.4) Assuming that K_τ has no local maximum for $0 < \tau < T$, then W_τ is strictly increasing and K_τ is strictly decreasing for $0 < \tau < T$ by **Lemma 3.3**, which immediately follows the uniqueness of τ^* under the assumptions. Assuming that K_τ has one local maximum at $\tau = \tau_0$ with $0 < \tau_0 < T$, then $\frac{dK_\tau}{d\tau} < 0$ for $\tau_0 < \tau < T$ and $\frac{dK_\tau}{d\tau} > 0$ for $0 < \tau < \tau_0$ since K_τ has no local minimum. **Lemma 3.3** shows $K_\tau < W_\tau$ for $\tau_0 - \varepsilon < \tau < T$ and thus $\tau^* < \tau_0$ if $\tau^* \in (0,T)$ exists. The existence of such τ^* follows from (11) since then $\frac{dI_\tau}{d\tau}$ changes the sign from positive to negative at $\tau = \tau^*$ and consequently $\frac{d^2 I_{\tau^*}}{d\tau^2} < 0$ follows because of continuity and smoothness of I_τ. Assume that this τ^* is the largest solution to $\frac{dI_\tau}{d\tau} = 0$ that locally maximizes I_τ. Then, since $g(W_\tau)$ is decreasing in τ, $\frac{d^2 I_{\tau^*}}{d\tau^2} < 0$ leads to

$$\mathrm{sgn}\left\{\frac{\mathrm{d}^2 I_\tau}{\mathrm{d}\tau^2}\right\} = \mathrm{sgn}\left\{R + \frac{c}{\gamma} - g(W_\tau)\right\} = -1 < 0 \tag{14}$$

for $\tau < \tau^*$ such that $W_\tau = K_\tau$. If there exist another $\tau = \tau_* < \tau^*$ that locally maximizes $\frac{dI_\tau}{d\tau}$, then there has to exist $\tilde{\tau}$ with $\tau_* < \tilde{\tau} < \tau^*$ that locally minimizes I_τ because of its continuity and smoothness. Such $\tilde{\tau}$ has to comply with both $\frac{dI_{\tilde{\tau}}}{d\tau} = 0$ and $\frac{d^2 I_{\tilde{\tau}}}{d\tau^2} > 0$, which contradicts (14). Uniqueness of $\tau^* \in (0,T)$ therefore follows under the assumptions.

Theorem 3.4 with the right equation of (8) immediately leads to the following proposition since $I \in C([0,T]) \cap C^2(0,T)$.

Proposition 3.5 $\lambda \equiv g(W_{\tau^*}) - \left(R + \frac{c}{\gamma}\right) > 0$ *under* ***Assumption 3.2*** *and* (11).

Proposition 3.5 leads to the following comparative statics results, which are numerically verified later as well.

Proposition 3.6: *Under* ***Assumption 3.2*** *and (11), the comparative statics results*

$$\frac{\partial \tau^*}{\partial \gamma} > 0, \quad \frac{\partial \tau^*}{\partial R} < 0, \quad \text{and} \quad \frac{\partial \tau^*}{\partial c} > 0 \tag{15}$$

follow where the last one is subject to sufficiently small c.

(Proof of Proposition 3.6) Differentiating both sides of $W_{\tau^*} = K_{\tau^*}$ with respect to each parameter and rearranging the resulting equation with the help of Leibnitz's rule (Appendix A of Yoshioka and Unami [20]) yields

$$\frac{\partial \tau^*}{\partial \gamma} = \frac{1}{\gamma(\gamma-1)}\lambda^{-1} > 0, \tag{16}$$

$$\frac{\partial \tau^*}{\partial R} = -c\left(1-\frac{1}{\gamma}\right)W_{\tau^*}^{-1}\lambda^{-1}\int_{\tau^*}^{T}(s-\tau^*)e^{-(R+c)(s-\tau^*)}W_s\mathrm{d}s < 0, \tag{17}$$

and

$$\frac{\partial \tau^*}{\partial c} = W_{\tau^*}^{-1}\lambda^{-1}\int_{\tau^*}^{T}\left[1-c\left(1-\frac{1}{\gamma}\right)(s-\tau^*)\right]e^{-(R+c)(s-\tau^*)}W_s\mathrm{d}s > 0 \tag{18}$$

where (18) follows if

$$1-c\left(1-\frac{1}{\gamma}\right)(s-\tau^*) > 1-c\left(1-\frac{1}{\gamma}\right)(T-\tau^*) > 1-c\left(1-\frac{1}{\gamma}\right)T > 0, \tag{19}$$

namely for sufficiently small c.

Remark 3.7

Proposition 3.6 immediately yields $\frac{\partial \tau^*}{\partial \alpha} > 0$, $\frac{\partial \tau^*}{\partial \beta} < 0$, and $\frac{\partial \tau^*}{\partial p} < 0$ since $\gamma = \frac{\alpha c}{\beta p}$.

Proposition 3.6 and **Remark 3.7** show that increasing the profit rate (α) or decreasing the farming cost (β or p) increases τ^* since harvesting (and selling) the more grown organisms results in more profitable. Increasing the harvesting rate c also increases τ^* since $\frac{\partial \tau^*}{\partial c} > 0$ at least for small c. On the other hand, increasing the mortality R results in smaller τ^* since only small number of individuals may remain near the end of the farming period.

A brief application of the present optimal stopping problem is provided focusing on an application to the commercially important fish *P. altivelis* in Japan, which are the main inland fishery resources in the country. The *P. altivelis* has an annual life history, which is reviewed in detail in the literature [15, 18] and the references therein. In each year, farming juveniles of *P. altivelis* in an aquaculture system starts in spring and they mature in summer around which harvesting opens. The harvesting ends in the coming autumn. Hii River Fishery Cooperatives in Shimane Prefecture, Japan that farms *P. altivelis* from May to October in each year measured the mean body weight of the individuals from May 7, 2015 ($t = 0$ (day)) to July 13, 2015 ($t = 55$ (day)) as shown in Fig. 1, which is fitted with the conventional Verhulst model [16]

$$\frac{\mathrm{d}W_t}{\mathrm{d}t} = bW_t\left(1-\frac{W_t}{a}\right) \tag{20}$$

with $W_0 = 9.9$ (g), $a = 76.7$ (g), and $b = 0.04$ (1/day) based on a standard nonlinear least squares method. Reasonable ranges of the parameters are $R, p, c = O(10^{-3})$ to $O(10^{-2})$ (1/day) and $\alpha, \beta = O(10^0)$ where the latter is set to be non-dimensional without the loss of generality. The terminal time is set as $T = 180$ (day). The optimal opening time τ^* is computed with the help of the classical four-stage Runge-Kutta method using (7) and (8).

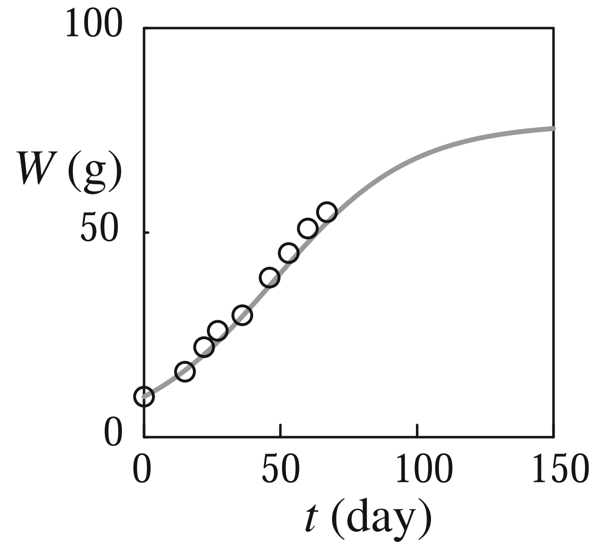

Fig. 1 Measured (circles) and identified W_t (curves) of farmed *P. altivelis*

Figure 2 shows profiles of W_τ and K_τ for $R = 0.005$ (1/day), $p = c = 0.01$ (1/day), $\alpha = 5$, and $\beta = 1$ **(Case 1)** and for $R = 0.005$ (1/day), $p = c = 0.02$ (1/day), $\alpha = 11$, and $\beta = 1$ **(Case 2)**. The parameters in **Case 1** verify (11) ($\tau^* \in (0,T)$) while those in **Case 2** do not ($\tau^* = 0$), which is consistent with **Theorem 3.4**. Fig. 2 directly implies (11).

Figures 3, 4 and 5 show the computed τ^* in the R-c phase space, p-c phase space, and α-β phase space, respectively. The results numerically confirm the comparative statics results of **Proposition 3.6**. It is finally noted that the current opening time τ^* that Hii River Fishery Cooperatives is adopting is 40 (day) to 50 (day). This is consistent with the computational results using the specified parameter values. To the authors' knowledge, this

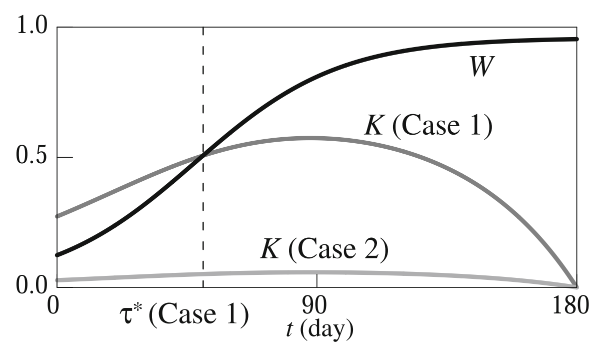

Fig. 2 W_τ and K_τ for $R = 0.005$ (1/day), $p = c = 0.01$ (1/day), $\alpha = 5$, and $\beta = 1$ (Case 1) and for $R = 0.005$ (1/day), $p = c = 0.02$ (1/day), $\alpha = 1.1$, and $\beta = 1$ (Case 2)

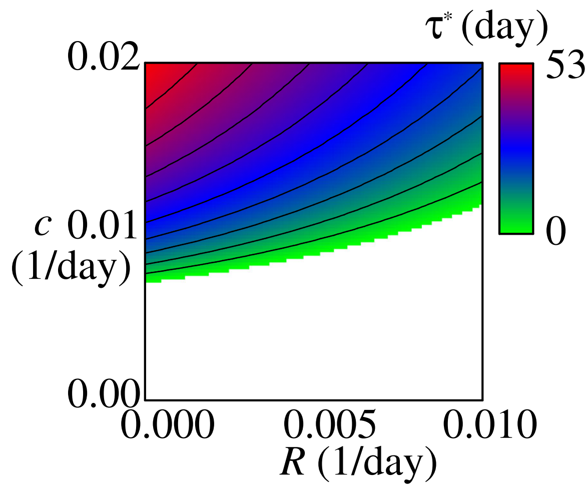

Fig. 3 Computed τ^* in the *R*-*c* phase space. $\tau^* = 0$ in the white area

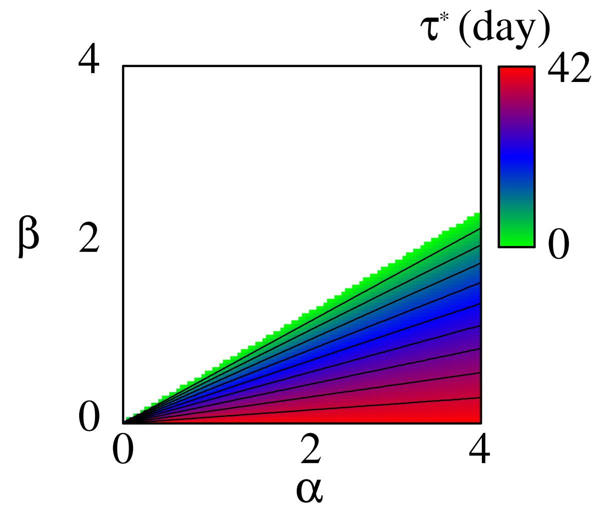

Fig. 5 Computed τ^* in the *α*-*β* phase space. $\tau^* = 0$ in the white area

kind of application of mathematical models to management of *P. altivelis* has not been performed so far.

4 Conclusions

As an application of mathematics to engineering problems, this paper formulated a simple optimal stopping problem to decide the opening time τ^* of harvesting farmed aquatic organisms in an aquaculture system. The existence and uniqueness of τ^* were mathematically analyzed. In addition, comparative statics of τ^* clearly revealed its dependence on the parameters on the farming environment. An application example of the present optimal stopping problem was finally provided with identified parameters.

Future research will extend the present model to a stochastic counterpart [2, 11] along with detailed mathematical analysis where uncertainties involved in the individual growth and harvesting rate are taken into account. Considering the investment and development cost [6] for expansion or abandonment of aquacultures systems is also a possible extension of the present mathematical model. Verifying the theoretically derived optimal opening times with real operations of aquacultures will be an important research topic as well. Addressing this topic would advance and sophisticate both mathematical science and fishery engineering.

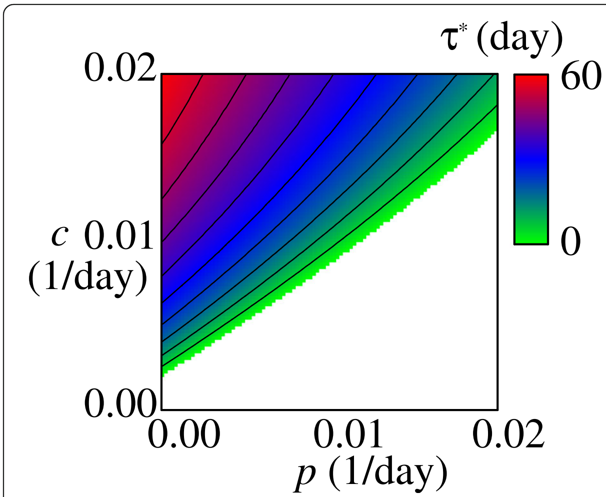

Fig. 4 Computed τ^* in the *p*-*c* phase space. $\tau^* = 0$ in the white area

Appendix

Derivation procedures of (4), (5), (6), and (7)

This appendix presents derivation procedures of (4), (5), (6), and (7) in the main text.

Firstly, I_τ can be rewritten as

$$\begin{aligned} I_\tau &= -\int_0^\tau e^{-Rs} W_s \mathrm{d}s + (\gamma-1)e^{-R\tau}\int_\tau^T e^{-(R+c)(s-\tau)} W_s \mathrm{d}s \\ &= -\int_0^\tau e^{-Rs} W_s \mathrm{d}s + (\gamma-1)e^{c\tau}\int_\tau^T e^{-(R+c)s} W_s \mathrm{d}s. \end{aligned} \tag{21}$$

Differentiating both sides of (21) with respect to τ yields

$$\begin{aligned} \frac{\mathrm{d}I_\tau}{\mathrm{d}\tau} &= \frac{\mathrm{d}}{\mathrm{d}\tau}\left(-\int_0^\tau e^{-Rs} W_s \mathrm{d}s\right) + \frac{\mathrm{d}}{\mathrm{d}\tau}\left((\gamma-1)e^{c\tau}\int_\tau^T e^{-(R+c)s} W_s \mathrm{d}s\right) \\ &= -e^{-R\tau}W_\tau + c(\gamma-1)e^{c\tau}\int_\tau^T e^{-(R+c)s} W_s \mathrm{d}s - (\gamma-1)e^{-R\tau}W_\tau \\ &= -\gamma e^{-R\tau}W_\tau + c(\gamma-1)e^{c\tau}\int_\tau^T e^{-(R+c)s} W_s \mathrm{d}s \\ &= \gamma e^{-R\tau}\left(-W_\tau + c\left(1-\frac{1}{\gamma}\right)\int_\tau^T e^{-(R+c)(s-\tau)} W_s \mathrm{d}s\right) \\ &= \gamma e^{-R\tau}(-W_\tau + K_\tau), \end{aligned} \tag{22}$$

which is (4) with K_τ defined in (6). Differentiating K_τ with respect to τ then yields

$$\begin{aligned}\frac{\mathrm{d}K_\tau}{\mathrm{d}\tau} &= \frac{\mathrm{d}}{\mathrm{d}\tau}\left[c\left(1-\frac{1}{\gamma}\right)\int_\tau^T e^{-(R+c)(s-\tau)}W_s\mathrm{d}s\right] \\ &= \frac{\mathrm{d}}{\mathrm{d}\tau}\left[c\left(1-\frac{1}{\gamma}\right)e^{(R+c)\tau}\int_\tau^T e^{-(R+c)s}W_s\mathrm{d}s\right] \\ &= (R+c)c\left(1-\frac{1}{\gamma}\right)e^{(R+c)\tau}\int_\tau^T e^{-(R+c)s}W_s\mathrm{d}s - c\left(1-\frac{1}{\gamma}\right)W_\tau \\ &= (R+c)K_\tau - c\left(1-\frac{1}{\gamma}\right)W_\tau\end{aligned} \tag{23}$$

which proves (7). Using (23) and (1), $\frac{\mathrm{d}^2 I_\tau}{\mathrm{d}\tau^2}$ can be directly calculated as

$$\begin{aligned}\frac{\mathrm{d}^2 I_\tau}{\mathrm{d}\tau^2} &= \frac{\mathrm{d}}{\mathrm{d}\tau}\left[\gamma e^{-R\tau}(-W_\tau + K_\tau)\right] \\ &= -R\gamma e^{-R\tau}(-W_\tau + K_\tau) + \gamma e^{-R\tau}\frac{\mathrm{d}}{\mathrm{d}\tau}(-W_\tau + K_\tau) \\ &= \gamma e^{-R\tau}\left(RW_\tau - RK_\tau - \frac{\mathrm{d}W_\tau}{\mathrm{d}\tau} + \frac{\mathrm{d}K_\tau}{\mathrm{d}\tau}\right) \\ &= \gamma e^{-R\tau}\left[RW_\tau - RK_\tau - W_\tau g(W_\tau) + (R+c)K_\tau - c\left(1-\frac{1}{\gamma}\right)W_\tau\right] \\ &= \gamma e^{-R\tau}\left[c(K_\tau - W_\tau) + \left(R + \frac{c}{\gamma} - g(W_\tau)\right)W_\tau\right],\end{aligned} \tag{24}$$

which is (5).

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